Variations of Hadron Masses and Matter Properties in Dense Nuclear Matter

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Abstract

Using a self-consistent quark model for nuclear matter we investigate variations of the masses of the non-strange vector mesons, the hyperons and the nucleon in dense nuclear matter (up to four times the normal nuclear density). We find that the changes in the hadron masses can be described in terms of the value of the scalar mean-field in matter. The model is then used to calculate the density dependence of the quark condensate in-medium, which turns out to be well approximated by a linear function of the nuclear density. Some relations among the hadron properties and the in-medium quark condensate are discussed.

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I INTRODUCTION

One of the most interesting future directions in nuclear physics may be to study how nuclear matter properties change as the environment changes. Forthcoming ultra-relativistic heavy-ion experiments are expected to give significant information on the strong interaction (i.e., QCD) in matter, through the detection of changes in hadronic properties [1, 2, 3]. In particular, the variations in hadron masses in the nuclear medium have attracted wide interest because such changes could be a signal of the formation of hot hadronic and/or quark-gluon matter in the energetic nucleus-nucleus collisions. Several authors have recently studied the vector-meson (ω, ρ, ϕ) masses using the vector dominance model [4], QCD sum rules [5] and the Walecka model (QHD) [6, 7, 8], and have reported that the mass decreases in the nuclear medium. There is also a proposal to look for such mass shifts at CEBAF [9].

In the approach based on QCD sum rules, the reduction of the mass is mainly due to the four-quark condensates and one of the twist-2 condensates, $\langle \bar{q}\gamma_\mu D_\mu q \rangle$. However, it has been suggested that there may be considerable, intrinsic uncertainty in the standard assumptions underlying the QCD sum-rule analyses [10]. On the other hand, in hadronic models like QHD [11], the main reason for the reduction in masses is the polarization of the Dirac sea [6, 7, 8], where the *antinucleons* in matter play a crucial role. However, from the point of view of the quark model, the strong excitation of *nucleon-antinucleon pairs* in medium is difficult to understand. It is clear that these two mechanisms are quite different.

Several years ago Guichon [12] proposed an entirely different model for nuclear matter, based on a mean-field description in which quarks (in nucleon bags) interact self-consistently with σ and ω mesons. The model has been refined by Fleck *et al.* [13] and the present authors. It provides a natural explanation of nuclear saturation and the right magnitude of the nuclear compressibility. We have used this model to investigate the nuclear structure functions [14], the properties of both nuclear and neutron matter [15] and the Okamoto-Nolen-Schiffer anomaly and isospin symmetry breaking in matter [16].

We argued that the response of the internal structure of the nucleon to its environment is vital to the understanding of, not only physics with momentum transfers of several GeV (eg., deep-inelastic scattering), but also physics at scale of a few MeV (eg., a violation of charge symmetry in nuclear medium). In Ref.[15], the relationship between the Guichon model (alias the quark-meson coupling (QMC) model in our papers) and QHD has also been clarified. Even though this model is extremely simple, the insights gained from it may help to reveal the essential physics [17]. Here we use the Guichon model to investigate variations of various hadronic properties, as well as the density dependence of the quark condensate in nuclear matter, as functions of the nuclear density up to four times normal nuclear density.

This paper is organized as follows. In Sec. II, the QMC model is introduced and applied to calculate the modification of the properties of the nucleon in nuclear matter. The masses of the vector mesons (ω, ρ) and the hyperons (Λ, Σ, Ξ) in matter are studied in Sec. III. We find that the change in the hadron masses can be described in terms of scalar mean-field values in medium. Some relationships among the hadron masses are given. Next, in Sec. IV, we use this model to study the density dependence of the quark condensate, which may be linked to a diverse range of nuclear phenomena. The density dependence of the quark condensate can be well fitted by a linear function of the nuclear density. Then, some connections among the hadron properties and the quark condensate in medium are discussed. Sec. V contains our conclusions.

II   MATTER PROPERTIES IN DENSE MEDIUM

II.1   The quark-meson coupling model

The QMC model treats nuclear matter as a collection of (nucleon) MIT bags, self-consistently bound by the exchange of scalar (σ) and vector (ω, ρ) mesons. We assume that nuclear matter with $N \neq Z$ is uniformly distributed, and that the mesons can be treated in mean-field approximation (MFA). Let the mean-field values for the σ, ω (the

time component) and ρ (the time component in the third direction of isospin) fields be $\bar{\sigma}$, $\bar{\omega}$ and $\bar{b}$, respectively. The quarks in a static spherical bag interact with those mean fields. The Dirac equation for a quark field, ψ_q, in a bag is then given by

$$[i\gamma \cdot \partial - (m_q - V_\sigma) - \gamma^0(V_\omega + \frac{1}{2}\tau_z V_\rho)]\psi_q = 0, \tag{1}$$

where $V_\sigma = g_\sigma^q \bar{\sigma}$, $V_\omega = g_\omega^q \bar{\omega}$ and $V_\rho = g_\rho^q \bar{b}$, with the quark-meson coupling constants, g_σ^q, g_ω^q and g_ρ^q. The bare quark mass is denoted by m_q and τ_z is the third Pauli matrix. The normalized, ground state for a quark is then given by

$$\psi_q(\vec{r}, t) = \mathcal{N}_q \exp[-i\epsilon_q t/R] \begin{pmatrix} j_0(x_q r/R) \\ i\beta_q \vec{\sigma} \cdot \hat{r} j_1(x_q r/R) \end{pmatrix} \frac{\chi_q}{\sqrt{4\pi}}, \tag{2}$$

where

$$\epsilon_q = \Omega_q + R(V_\omega \pm \frac{1}{2}V_\rho), \text{ for } \begin{pmatrix} u \\ d \end{pmatrix} \text{ quark}, \tag{3}$$

$$\mathcal{N}_q^{-2} = 2R^3 j_0^2(x_q)[\Omega_q(\Omega_q - 1) + Rm_q^\star/2]/x_q^2, \tag{4}$$

$$\beta_q = \sqrt{(\Omega_q - Rm_q^\star)/(\Omega_q + Rm_q^\star)}, \tag{5}$$

with $\Omega_q = \sqrt{x_q^2 + (Rm_q^\star)^2}$ and χ_q the quark spinor. The effective quark mass, $m_q^\star$, is defined by

$$m_q^\star = m_q - V_\sigma, \tag{6}$$

for both u and d quarks. The linear boundary condition, $j_0(x_q) = \beta_q j_1(x_q)$, at the bag surface determines the eigenvalue, x_q.

Using the SU(6) spin-flavor wave function for the nucleon, the nucleon energy is given by $E_{bag}^N + 3V_\omega \pm \frac{1}{2}V_\rho$ for $\begin{pmatrix} p \\ n \end{pmatrix}$, where the bag energy is

$$E_{bag}^N = \frac{\sum_q n_q \Omega_q - z_N}{R} + \frac{4}{3}\pi BR^3, \tag{7}$$

with B the bag constant and z_N a phenomenological parameter accounting for a multitude of corrections, including zero-point motion. Here n_q is the number of quarks in the nucleon. To correct for spurious c.m. motion in the bag [18] the mass of the nucleon at rest is taken to be

$$M_N = \sqrt{(E_{bag}^N)^2 - \sum_q n_q(x_q/R)^2}. \tag{8}$$

The effective nucleon mass, $M_N^\star$, in nuclear matter is given by minimizing eq.(8) with respect to R.

Table 1: $B^{1/4}$ and z_N for some bag radii ($m_0 = 5$ MeV).

$R_0(fm)$	0.6	0.8	1.0
$B^{1/4}$ (MeV)	187.7	157.2	136.1
z_N	2.038	1.640	1.169

To see the sensitivity of our results to the bag radius of the free nucleon, R_0, we choose the current quark mass, $m_q = m_0 (= m_u = m_d) = 5$ MeV, and vary the parameters, B and z_N, to fit the nucleon mass at 939 MeV with R_0 ($= 0.6, 0.8, 1.0\ fm$). The values of $B^{1/4}$ and z_N are listed in Table 1.

For infinite nuclear matter we take the Fermi momenta for protons and neutrons to be k_{F_j} ($j = p$ or n). This is defined by $\rho_j = k_{F_j}^3/(3\pi^2)$, where ρ_j is the density of protons or neutrons, and the total baryon density, ρ_B, is then given by $\rho_p + \rho_n$.

Since we want to calculate the variations in the masses of the ω and ρ mesons, we suppose that both mesons are also described by the MIT bag model in the scalar mean-field. To fit the free masses, $m_\omega = 783$ MeV and $m_\rho = 770$ MeV, we introduce new z-parameters for the two mesons, z_ω and z_ρ. We then find that $z_{\omega(\rho)} = 0.7840(0.8061)$, $0.4812(0.5160)$, $0.1198(0.1680)$ for $R_0 = 0.6, 0.8, 1.0\ fm$, respectively. Unfortunately, in this model it is hard to deal with the density dependence of the σ meson in medium because it couples strongly to the pseudoscalar (π) channel, which requires a direct treatment of chiral symmetry in medium [19]. That is beyond the scope of this study. Though one might expect the σ-meson mass in medium to be less than the free one (see, eg., Ref. [6]), we shall keep the free value, 550 MeV, even in medium.

The ω field is now determined by baryon number conservation as $\bar{\omega} = g_\omega \rho_B/m_\omega^{\star 2}$, where $g_\omega = 3g_\omega^q$ and $m_\omega^\star$ is the effective ω-meson mass, and the ρ mean-field by the

difference in proton and neutron densities, $\rho_3 = \rho_p - \rho_n$. On the other hand, the scalar mean-field is given by a self-consistency condition (SCC) [11, 15]. Since the ρ field value is $\bar{b} = g_\rho \rho_3 / (2 m_\rho^{\star 2})$, where $g_\rho = g_\rho^q$, the total energy per nucleon, E_{tot}, can be written

$$E_{tot} = \frac{2}{\rho_B (2\pi)^3} \sum_{j=p,n} \int^{k_{F_j}} d\vec{k} \sqrt{M_N^{\star 2} + \vec{k}^2} + \frac{m_\sigma^2}{2\rho_B} \bar{\sigma}^2 + \frac{g_\omega^2}{2m_\omega^{\star 2}} \rho_B + \frac{g_\rho^2}{8 m_\rho^{\star 2} \rho_B} \rho_3^2, \qquad (9)$$

where $m_\rho^\star$ is the effective ρ-meson mass in medium. Then, the SCC for the σ field is given by

$$\bar{\sigma} = -\frac{2}{(2\pi)^3 m_\sigma^2} \left[\sum_{j=p,n} \int^{k_{F_j}} d\vec{k} \frac{M_j^\star}{\sqrt{M_j^{\star 2} + \vec{k}^2}} \right] \left(\frac{\partial M_N^\star}{\partial \bar{\sigma}} \right)_R. \qquad (10)$$

Using Eqs.(7) and (8), we find

$$\left(\frac{\partial M_N^\star}{\partial \bar{\sigma}} \right)_R \equiv -g_\sigma C_N(\bar{\sigma}), \qquad (11)$$

where $g_\sigma = 3 g_\sigma^q$ and C_N is the quark-scalar density in the nucleon:

$$C_N(\bar{\sigma}) = \left(\frac{E_{bag}^N}{M_N^\star} \right) \left[\left(1 - \frac{\Omega_q}{E_{bag}^N R} \right) S_N + \frac{m_q^\star}{E_{bag}^N} \right], \qquad (12)$$

with

$$S_N = \int_R d\vec{r} \bar{\psi}_q \psi_q = \frac{\Omega_q / 2 + R m_q^\star (\Omega_q - 1)}{\Omega_q (\Omega_q - 1) + R m_q^\star / 2}. \qquad (13)$$

The value of the quark-scalar density is about 0.4 at $\rho_B = 0$, and it decreases monotonically to about 0.2 at the normal nuclear density, $\rho_0 = 0.17 fm^{-3}$. The dependence of the scalar density on the bag radius is not strong (see Ref.[15]).

II.2 Coupling constants and nuclear matter properties

We determine the coupling constants, g_σ^2 and g_ω^2, so as to fit the binding energy (-16 MeV) at the saturation density, ρ_0, for symmetric nuclear matter. Furthermore, the ρ-meson coupling constant is used to reproduce the bulk symmetry energy, 33.2 MeV. The values of the coupling constants are listed in Table 2. In the last two columns, the relative changes (with respect to the values at zero density) of the bag radius and the lowest eigenvalue are shown. The present model gives a good value for the nuclear compressibility – around 200 MeV. If we take a heavier current-quark mass, the effective nucleon mass and the nuclear

Table 2: The coupling constants and calculated properties of equilibrium matter at the saturation density ($m_0 = 5$ MeV). The effective nucleon mass, $M_N^\star$, and the nuclear compressibility, K, are quoted in MeV.

$R_0(fm)$	$g_\sigma^2/4\pi$	$g_\omega^2/4\pi$	$g_\rho^2/4\pi$	$M_N^\star$	K	$\frac{\delta R}{R_0}$	$\frac{\delta x}{x_0}$
0.6	18.79	1.326	4.923	838	223	-0.03	-0.07
0.8	20.63	1.016	5.014	850	205	-0.02	-0.09
1.0	21.09	0.871	5.045	855	196	-0.01	-0.12

Figure 1: Scalar mean-field values. The dotted, solid and dashed curves are, respectively, for $R_0 = 0.6$, 0.8 and 1.0 fm.

compressibility are, respectively, reduced and enhanced. The strength of the scalar mean-field, V_σ, in medium is shown in Fig.1. At small density it is well approximated by a linear function of the density (see also Eq.(40) in Ref.[15]):

$$V_\sigma \approx 140 \text{ (MeV) } \left(\frac{\rho_B}{\rho_0}\right). \tag{14}$$

Figure 2: Effective nucleon mass in symmetric nuclear matter. The curves are labelled as in Fig.1.

The dependence of the effective nucleon mass on the nuclear density is shown in Fig.2. It decreases as the density goes up, and it behaves like constant at large density. We find that the effective nucleon mass at small density is approximately given by

$$\frac{M_N^\star}{M_N} \simeq 1 - 0.14 \left(\frac{\rho_B}{\rho_0}\right), \tag{15}$$

which is identical to the model independent result derived using QCD sum-rules [20].

Figure 3: The ratio of the axial-vector coupling constant of the nucleon in medium to that in free space. The curves are labelled as in Fig.1.

Figure 4: The ratio of the magnetic moment of the proton in medium to that in free space. The curves are labelled as in Fig.1.

In Fig.3 the density dependence of the axial-vector coupling constant of the nucleon is illustrated. It decreases as the density increases. On the other hand, the magnetic moment of the proton in symmetric nuclear matter increases with density as shown in Fig.4. The calculation of these quantities is standard [21] (c.m. and recoil corrections are not included [22]). It is rather easy to understand why $g_A^\star$ and $\mu_N^\star$ tend to decrease and increase, respectively, with density: in the bag model these quantities involve integrals over the upper and lower components of the quark wave function. Because the attractive scalar potential effectively decreases the quark mass it makes the solution more relativistic – i.e., the lower component of the wave function is enhanced. This fact leads, respectively, to the decrease and increase of $g_A^\star$ and $\mu_N^\star$ in medium. Given the simplicity of this argument we find the conclusion that the magnetic moments of the other hadrons should increase quite compelling.

At small density, we can expand $g_A^\star$ and $\mu_N^\star$ in terms of ρ_B. If we take the quark to be massless for simplicity, and ignore the dependence of $g_A^\star$ and $\mu_N^\star$ on the change of the bag radius in medium, they are expressed as

$$\frac{g_A^\star}{g_A} \simeq 1 - \frac{4x_0^3 - 12x_0^2 + 10x_0 - 3}{2x_0^2(x_0 - 1)^2}(RV_\sigma) \simeq 1 - 0.09\left(\frac{\rho_B}{\rho_0}\right), \qquad (16)$$

and

$$\frac{\mu_N^\star}{\mu_A} \simeq 1 - \frac{4x_0^3 - 16x_0^2 + 17x_0 - 6}{2x_0(x_0 - 1)^2(4x_0 - 3)}(RV_\sigma) \simeq 1 + 0.1\left(\frac{\rho_B}{\rho_0}\right), \tag{17}$$

where $x_0 = 2.04$, $R = 0.8$ fm and Eq.(14) is used. These formulae can roughly reproduce both the ratios at small density. Furthermore, we have calculated the charge radius of the proton in medium, and we find that the change is within a few per cent even at $\rho_B \sim 4\rho_0$.

Here we must add a caution that the contribution of meson exchange currents (MEC) to $g_A^\star$, $\mu_N^\star$, etc. would be considerable in medium, and, hence, we should treat the whole problem including MEC to get the final results for the changes in these quantites. This is obviously beyond the scope of the present work.

Finally, we record that the variations of the above nucleon properties depend only very weakly on the proton fraction, f_p, which is defined as ρ_p/ρ_B, because the ρ meson does not play a role in determining the nucleon structure in medium (see Eqs.(6) $\sim$ (8)). Further investigations concerning the nucleon properties, the equation of state for matter, etc. can be found in Ref.[14, 15, 16].

III HADRON MASSES IN MEDIUM

Now we are in a position to present our main results. In Fig.5, the ratio of the effective

Figure 5: The ratio of the vector-meson mass in symmetric nuclear matter to the free mass. The curves are labelled as in Fig.1.

vector-meson mass, $m_v^\star$, to that in free space is shown as a function of the density. Since the difference between the effective ω- and ρ-meson masses at the same density is very small, we show only one curve for both mesons in the figure. As the density increases the vector-meson mass decreases (as several authors have previously noticed [4, 5, 6, 8]), and seems to become flat like the effective nucleon mass. The mass reduction can be well expressed by a linear form at small density:

$$\frac{m_v^\star}{m_v} \simeq 1 - 0.09 \left(\frac{\rho_B}{\rho_0} \right). \tag{18}$$

The reduction factor, 0.09, is somewhat smaller than that predicted by the QCD sum rules [8]. In this model the reduction in the mass is basically caused by the attractive scalar mean-field in medium, and this origin is clearly different from that in QHD [6, 8], in which the essential mechanism is the vacuum polarization due to the excitation of *nucleon-antinucleon pairs* in medium.

It is possible to calculate masses of other hadrons in this model. In particular, there is considerable interest in studying the properties of hyperons in medium – eg. Λ, Σ and Ξ. For the hyperons themselves we again use the MIT bag model, including the c.m. corrections (see Eq.(8)). We assume that the strange quark does not couple to the σ meson in MFA, and that the addition of a single hyperon to nuclear matter of density ρ_B does not alter the values of the scalar and vector mean-fields. The mass of the strange quark, m_s, is determined so as to reproduce the mass splitting between the nucleon and the average of Λ and Σ hyperons, $M_{H_{av}} (= 1154 \text{ MeV})$, in free space. We then find m_s

$= 355.5, 358.1, 354.0$ MeV for $R_0 = 0.6, 0.8, 1.0$ fm, respectively. Using these values of the strange-quark mass, we have calculated the average mass of Λ and Σ, $M^\star_{H_{av}}$, and the mass of Ξ, $M^\star_\Xi$, in symmetric nuclear matter.

Figure 6: The ratio of the average mass of Λ and Σ in medium to that in free space. The curves are labelled as in Fig.1.

In Fig.6, the average mass, $M^\star_{H_{av}}$, in medium is illustrated. The density dependence of $M^\star_{H_{av}}$ is very similar to the cases of the vector meson and the nucleon. The ratio is again well described by a linear function at small density:

$$\frac{M^\star_{H_{av}}}{M_{H_{av}}} \simeq 1 - 0.08 \left(\frac{\rho_B}{\rho_0} \right). \tag{19}$$

Note that the reduction factor in the mass formula is almost the same value as in Eq.(18).

Figure 7: The ratio of the Ξ mass in medium to that in free space. The curves are labelled as in Fig.1.

The effective Ξ-hyperon mass in medium is shown in Fig.7. The ratio again behaves like the case of $M^\star_{H_{av}}$, but an approximate form for $M^\star_\Xi / M_\Xi$ at low density is given by

$$\frac{M^\star_\Xi}{M_\Xi} \simeq 1 - 0.04 \left(\frac{\rho_B}{\rho_0} \right), \tag{20}$$

where the reduction factor is about half of those in Eqs.(18) and (19).

We summarize the effective masses of the hadrons in medium in Fig.8. As one can see from the figure, the reduction in both the vector-meson mass and the average mass of Λ and Σ are about twice that in the Ξ-hyperon mass over a wide range of ρ_B, while the reduction in the nucleon is about three times that in the Ξ. One can also see this fact from the reduction factors in Eq.(15) and Eqs.(18) $\sim$ (20). It is rather easy to understand

Figure 8: The ratios of the effective masses of the hadrons in symmetric nuclear matter to those in free space ($R_0 = 0.8 \ fm$). The dashed, solid, dotted and dot-dashed curves are for Ξ, $(\Lambda + \Sigma)/2$, the vector (ω, ρ) meson and the nucleon, respectively.

why such a relationship holds among the hadron masses: the nucleon consists of three non-strange quarks, which couple to the scalar mean-field in medium, while the vector mesons, and the Λ and Σ hyperons, involve just two non-strange quarks. The Ξ hyperon consists of only one non-strange quark and two strange quarks. Since the strange quark does not feel the scalar mean-field, its mass is not altered in medium. The change in the hadron mass is therefore roughly determined by the number of non-strange quarks, n_0, and the strength of the scalar mean-field, V_σ. We can then find an approximate form of the hadron mass in symmetric nuclear matter:

$$\frac{M^\star_{hadron}}{M_{hadron}} \simeq 1 - \text{ const } \times n_0 V_\sigma \text{ (MeV)}, \tag{21}$$

where const $\simeq 3.2 \times 10^{-4} \text{MeV}^{-1}$. Note that this formula is not too sensitive to the bag radius, and that it can reproduce the hadron masses reasonably well over the range of ρ_B up to about $3\rho_0$. From Eq.(21) we expect the following relations among the hadron masses:

$$\left(\frac{M^\star_N}{M_N}\right) \approx \left(\frac{M^\star_\Xi}{M_\Xi}\right)^3, \quad \left(\frac{m^\star_v}{m_v}\right) \approx \left(\frac{M^\star_\Lambda}{M_\Lambda}\right) \approx \left(\frac{M^\star_\Sigma}{M_\Sigma}\right) \approx \left(\frac{M^\star_\Xi}{M_\Xi}\right)^2. \tag{22}$$

Furthermore, using Eqs.(16) and (17), $g^\star_A$, $\mu^\star_N$ and $m^\star_v$ at small density could be linked as

$$\left(\frac{m^\star_v}{m_v}\right) \approx \left(\frac{g^\star_A}{g_A}\right) \approx \left(\frac{\mu^\star_N}{\mu_N}\right)^{-1}. \tag{23}$$

Finally, we record that the proton-fraction dependence of the effective masses we have studied in this section is again very weak.

IV QUARK CONDENSATES IN MEDIUM

Having shown that the QMC model provides an interesting description of the hadron mass in nuclear matter we now apply it to study the behavior of the quark condensates in medium. The quark condensates are very important parameters in the QCD sum rules, and it is believed that they are linked to a wide range of nuclear phenomena, including the effective hadron mass in medium [3, 23].

The difference of the quark condensate in matter, $Q(\rho_B)$, and that in vacuum, $Q(0)$, is given through the Hellmann-Feynman theorem [15, 16, 17, 24]:

$$
\begin{aligned}
Q(\rho_B) - Q(0) &= \frac{1}{2}\frac{\partial \mathcal{E}}{\partial m_q}, \\
&= \frac{\partial \mathcal{E}}{\partial M_N^\star}\frac{dM_N^\star}{dm_q} + \sum_{j=mesons}\frac{\partial \mathcal{E}}{\partial m_j^\star}\frac{dm_j^\star}{dm_q} + \sum_j \frac{\partial \mathcal{E}}{\partial g_j}\frac{dg_j}{dm_q} + ...,
\end{aligned}
\tag{24}
$$

where $\mathcal{E} = \rho_B E_{tot}$.

Using the SCC, one finds

$$
\frac{\partial \mathcal{E}}{\partial M_N^\star} = \frac{1}{C_N(\bar\sigma)}\left(\frac{m_\sigma}{g_\sigma}\right)^2 (g_\sigma\bar\sigma),
\tag{25}
$$

$$
\frac{dM_N^\star}{dm_q} = 3C_N(\bar\sigma)\left(1 - \frac{dV_\sigma}{dm_q}\right),
\tag{26}
$$

and

$$
\frac{dm_v^\star}{dm_q} = 2C_v(\bar\sigma)\left(1 - \frac{dV_\sigma}{dm_q}\right),
\tag{27}
$$

where $v = \omega$ or ρ, and C_v is the quark-scalar density in the vector meson given by Eq.(12) (using the variables for the vector meson instead of those for the nculeon). Then, using the parametrization for the derivative of the σ-meson mass with respect to the quark mass [24], $\frac{dm_\sigma}{dm_q} = \frac{\sigma_N m_\sigma}{m_q M_N}$, where $\sigma_N(= 3m_q C_N(0))$ is the nucleon σ term [25], and the Gell-Mann–Oakes–Renner relation, the ratio of the quark condensate in medium to that in vacuum is written by a rather lengthy formula:

$$
\frac{Q(\rho_B)}{Q(0)} = 1 - \frac{\sigma_N}{C_N(0)m_\pi^2 f_\pi^2}\left[\left(\frac{m_\sigma}{g_\sigma}\right)^2 (g_\sigma\bar\sigma) + A_1\bar\sigma^2 + A_2\rho_B^2 + A_3\rho_3^2\right],
\tag{28}
$$

where m_π is the pion mass (138 MeV), f_π the pion decay constant (93 MeV) and the coefficients, $A_1 \sim A_3$, are given by

$$A_1 = C_N(0)\frac{m_\sigma^2}{M_N} - \frac{1}{6}\left(\frac{m_\sigma}{g_\sigma}\right)^2 \frac{dg_\sigma^2}{dm_q}, \tag{29}$$

$$A_2 = \frac{1}{6m_\omega^{\star 2}}\frac{dg_\omega^2}{dm_q} - \frac{2C_\omega(\bar\sigma)g_\omega^2}{3m_\omega^{\star 3}}\left(1 - \frac{dV_\sigma}{dm_q}\right), \tag{30}$$

and

$$A_3 = \frac{1}{24m_\rho^{\star 2}}\frac{dg_\rho^2}{dm_q} - \frac{2C_\rho(\bar\sigma)g_\rho^2}{12m_\rho^{\star 3}}\left(1 - \frac{dV_\sigma}{dm_q}\right). \tag{31}$$

We use $\sigma_N = 45$ MeV [25]. The derivatives of the scalar mean-field and the coupling constants with respect to the quark mass are calculated numerically – eg., the coupling constants are approximately given by quadratic or linear functions of the quark mass: for $R_0 = 0.8\ fm$, $g_\sigma^2 = 269.6 - 2.150m_0 + 0.01438m_0^2$, $g_\omega^2 = 11.97 + 0.1616m_0$, $g_\rho^2 = 63.38 - 0.076m_0$.

Figure 9: The ratio of the quark condensate in medium to that in vacuum ($m_0 = 5$ MeV and $R_0 = 0.8\ fm$). The solid and dotted curves are, respectively, for $f_p = 0.5$ and 0.

In Fig.9 we show the ratio of the quark condensate in medium to that in vacuum. As one can see, the density dependence of the quark condensate can be well fitted by a linear function of the density :

$$\frac{Q(\rho_B)}{Q(0)} \simeq 1 - \binom{0.36}{0.34}\left(\frac{\rho_B}{\rho_0}\right), \tag{32}$$

where the reduction factors, $\binom{0.36}{0.34}$, are for the proton fractions, $f_p = \binom{0.5}{0}$, respectively. The value of 0.36 agrees with the model-independent prediction of Cohen *et al.* [24] for symmetric nuclear matter. Note that the ratio is not sensitive to the bag radius. If we take a heavier current quark mass (eg., $m_0 = 10$ MeV), the reduction in the ratio becomes smaller (about 1 % at $\rho_B = 2\rho_0$) than the present case.

Taking up the terms of $\mathcal{O}(\rho_B)$ in Eq.(24) to see the behavior at low density, the ratio is given by

$$\frac{Q(\rho_B)}{Q(0)} \simeq 1 - \frac{3\sigma_N}{C_N(0)m_\pi^2 f_\pi^2}\left(\frac{m_\sigma}{g_\sigma}\right)^2 V_\sigma(\bar{\sigma}). \tag{33}$$

Using Eq.(14), $g_\sigma^2 \simeq 260$ and $C_N(0) \simeq 0.37$, Eq.(33) can reproduce Eq.(32). On the other hand, since the ratio of the hadron mass in medium to that in free space is roughly described by Eq.(21), the ratio of the quark condensates could be linked to the ratio of the hadron masses at low density:

$$\frac{M_{hadron}^\star}{M_{hadron}} \simeq 1 - 0.124 \times n_0\left(1 - \frac{Q(\rho_B)}{Q(0)}\right). \tag{34}$$

This relation suggests that the ratio of the effective hadron mass to the free one does not always scale as $\left(\frac{Q(\rho_B)}{Q(0)}\right)^{1/3}$ [3]. At small density we could expect Eq.(22), (23) and

$$\left(\frac{m_v^\star}{m_v}\right) \approx \left(\frac{Q(\rho_B)}{Q(0)}\right)^{1/4}, \tag{35}$$

rather than the naive $\left(\frac{Q(\rho_B)}{Q(0)}\right)^{1/3}$ scaling.

V CONCLUSION

We have applied the quark-meson coupling (QMC) model to investigate the density dependence of the properties of hadrons and the quark condensates in dense nuclear matter (up to four times the normal nuclear density). We have calculated not only the variations in the nucleon properties in medium but also those in the masses of the vector (ω, ρ) mesons and the hyperons (Λ, Σ, Ξ). As several authors have suggested [4, 5, 6, 8], the hadron mass is reduced due to the change of the scalar mean-field in medium. In the present model the hadron mass can be related to the number of non-strange quarks and the strength of the scalar mean-field. The hadron masses are simply connected to one another, and the relationship among them is given by Eq.(22), which is effective over a wide range of the nuclear density. Furthermore, the ratio of the quark condensate in medium to that in vacuum can be related to the vector-meson mass and the nucleon properties – eg., $g_A^\star$ and $\mu_N^\star$ – in medium.

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Finally, we would like to give a few caveats concerning the present calculation. The basic idea of the model is that the mesons are locally coupled to the quarks. This is certainly common for pions in models like the chiral or cloudy bag [26], but may be less justified for heavier mesons like the vector mesons, which are obviously not collective states. The model also omits the effect of short-range correlations among the quarks. At very high density these would be expected to dominate. Furthermore, the pionic cloud of the hadron [26] should be considered explicitly in any truly quantitative study of the hadron properties in medium. As we mentioned at the end of Sec. II, the contribution of meson exchange currents in medium is also important and should be considered in a consistent manner. Finally, we note that subtleties such as scalar-vector mixing in medium and the splitting between longitudinal and transverse masses of the vector mesons [7] have been ignored in the present mean-field study. Although the former appears to be quite small in QHD the latter will certainly be important in any attempt to actually measure the mass shift.

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